



# Library of Mathematical Transforms over Algorithms of Spectral-Analytical Data Processing



Applications in data analysis and pattern recognition



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## Methods

- **Generalized Spectral-Analytical Method (GSAM) as extrapolation of classical Fourier series to a wider class of basic approximating functions**
- **Parallel, recurrent and iterative algorithms of data analysis and transforms**

## Applications

- **Processing of biomechanical experiments data, the control of human state by means of body movements analysis (possible use for driver state recognition)**
- **Data analysis and diagnosis in electrophysiology (gastrology, cardiology, encephalography)**
- **Analysis of magnetic encephalography data – the brain functional charting and pathology states control**
- **Spectral methods of biological macromolecules analysis - genetic sequences, primary and spatial structures of proteins**
- **Data analysis and recognition in the space surveillance systems**
- **Monitoring and Diagnostics in the Technical imaging**
- **Data analysis, modeling and forecasting in the transportation acoustic ecology**

## Cooperation

- **Russian Academy of Sciences –**
  - ✓ **Institute of Biophysics**
  - ✓ **Institute of Protein Researches**
  - ✓ **Dorodnicyn Computing Centre**
  - ✓ **Blagonravov Institute of Machines Science**
  - ✓ **Kotel'nikov Institute of Radio-engineering and Electronics**
- **Lomonosov Moscow State University**
- **Federal Space Agency**
- **Russia Tunneling Association**
- **Moscow SubWay Administration**
- **Prague Technical University**
- **New York University**
- **Berlin Technical University**

## Grants

- **Russian Foundation for Basic Researches**
- **U.S. Civilian Research and Development Foundation**

## Publications

- F. F. Dedus, S. A. Makhortykh, M. N. Ustinin, and A. F. Dedus. Generalized Spectral-Analytic Method for Data Processing,” in Problems in Image Analysis and Pattern Recognition (Mashinostroenie, Moscow, 1999) [in Russian], 356 p.
- F.F.Dedus, S.A.Makhortykh and M.N.Ustinin. Generalized Spectral-Analytic Method in Information Processing Problems. *Pattern Recognition and Image Analysis*, 2002, [vol.12, N 4, pg.429-437](#).
- A. N. Pankratov, S. A. Makhortykh, et al. Spectral Analysis for Identification and Visualization of Repeats in Genetic Sequences. *Pattern Recognition and Image Analysis*, 2009, Vol. 19, No. 4, pp. 687-692.
- S. Makhortykh, E. Lyzhko. Sources localization for brain biomagnetic activity. *Review of Applied Physics (RAP)*, 2014, vol.3, pg.25-28.
- S.A.Kostarev, S.A.Makhortykh, S.A.Rybak. Calculations of ground vibrations induced by underground sources: analytical and numerical approaches, In: Noise and vibration from high-speed trains. London, Thomas Telford Publishing, 2001, pg. 397-422.

# Laboratory of Data Processing

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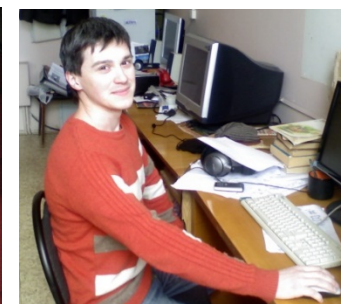
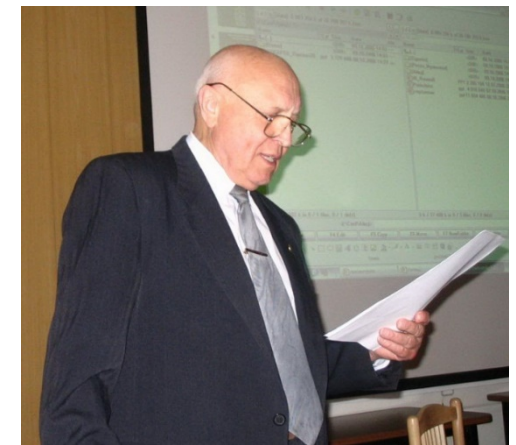
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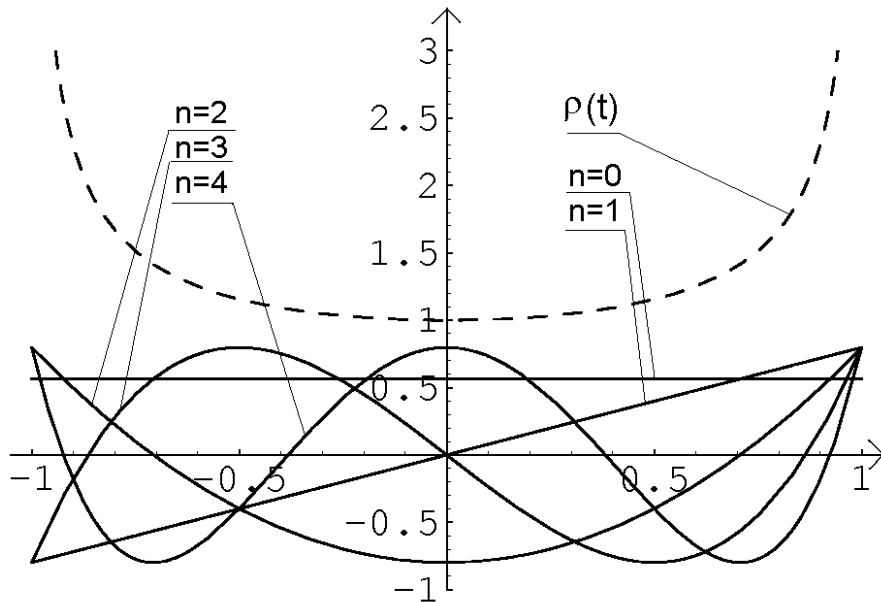
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# Classical orthogonal polynomials

**Table 1.** Classical orthogonal polynomials

No.	Polynomial	Symbol	General expression	Weight $p(x)$	Limits of existence		
					Lower	Upper	
1	Jacobi or hypergeometric	$P_n^{\alpha\beta}(x)$	$P_n^{\alpha\beta}(x) = \frac{1}{2^n} \sum_{k=0}^n C_n^k \frac{\Gamma(\alpha+n+1)\Gamma(\beta+n+1)}{\Gamma(\alpha+k+1)\Gamma(\beta+n+k+1)} (x-1)^k (x+1)^{n-k}$	$(1-x)^\alpha(1+x)^\beta$ $\alpha > -1 \quad \beta > -1$	-1	+1	
2	Gegenbauer of ultraspherical	$C_n^\sigma(x)$	$C_n^\sigma(x) = \sum_{k=0}^n \frac{(-1)^k \Gamma(\alpha+n-k)}{\Gamma(k+1)\Gamma(n-2k+1)} (2x)^{n-2k}$	$(1-x^2)^{\sigma-0.5}$ $\alpha = \beta = \sigma - 0.5$	-1	+1	
3	Chebyshev of the first kind	$T_n(x)$	$T_n(x) = \frac{2^n n!}{(2n)!} \sqrt{x^2-1} \frac{d^n}{dx^n} [(x^2-1)^{n-2k}]$	$(1-x^2)^{-0.5}$ $\alpha = \beta = -0.5$	-1	+1	
4	Chebyshev of the second kind	$U_n(x)$	$U_n(x) = \frac{2^n (n+1)!}{(2n+1)! \sqrt{x^2-1}} \frac{d^n}{dx^n} [(x^2-1)^{n+0.5}]$	$(1-x^2)^{0.5}$ $\alpha = \beta = 0.5$	-1	+1	
				$\frac{d^n}{dx^n} [(x^2-1)^n]$	1 $\alpha = \beta = 0$	-1	+1
				$\int_0^\infty C_{n+\alpha}^{n-k} \frac{(-x)^k}{k!}$	$x^\alpha e^{-x}$	0	$\infty$
				$\int_0^\infty C_n^{n-k} \frac{(-x)^k}{k!}$	$e^{-x}$	0	$\infty$
				$\int_0^\infty 1)^n e^{x^2} \frac{d}{dx^n} (e^{-x^2})$	$e^{-x^2}$	$\infty$	$\infty$



**Table 2.** Classical orthogonal bases of a discrete argument

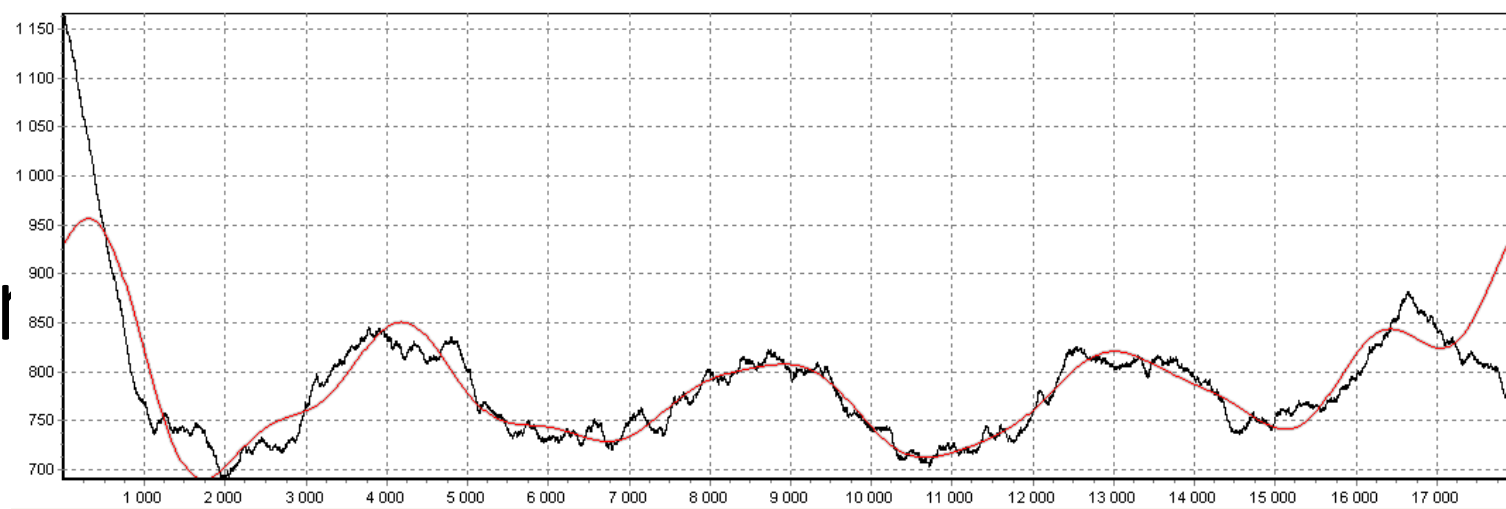
No.	Orthogonal basis	Symbol	Orthogonal Bases		Saltus function (weight function)	Limits of existence			
						Lower	Upper		
1	Chebyshev	$\theta_n(t)$	$n! \Delta \left[ \binom{t}{n} \binom{t-N}{n} \right]$		1	0	$N-1$		
2	Kravchuk's polynomials	$G_n(t; p, q)$	continuous	discrete	$\binom{N}{t} p^t q^{N-t}$	0	$N$		
			$\frac{(-1)^n}{n!} \left[ \frac{p^t q^{N-t+n}}{(t-n)! (N-t)!} \right] \Delta^n$		$p > 0, q > 0, p+q=1$				
		$\alpha(x)y' + \tau(x)y' + \lambda_k y = 0$	Polynomial orthogonal bases						
			Bases of classical polynoms						
4	Charlier's polynomials		$\sum_{i=0}^n (-1)^i \frac{n! t! m^i}{i! (n-i)! (t-i)!}$		$m > 0$	0	$\infty$		
			Bases of classical polynoms						
5	Charlier's		$\sqrt{t!} m^{i-0.5t} \exp(-\frac{1}{2m})$		$m > 0$		$\infty$		
6	Meixner		Hermit $H_n$	Jacobi $P_n^{(\alpha, \beta)}$	Sonin-Laguerre $L_n^{(\omega)}$	Kravchuk $k_n^{(p)}$	Han $h_n^{(\alpha, \beta)}$	Meixner $m_n^{(\lambda, \mu)}$	Charle $c_n^{(\mu)}$
7	Meixner's functions		Gegenbauer $C_n^{(\lambda)}$	Laguerre $L_n$			Tschebyshev (discrete) $\theta_n^{(\lambda)}$		
8	Hahn's (g byshev)		Legendre $P_n$	Tschebyshev I $T_n$	Tschebyshev II $U_n$	$0 < c < 1, \beta > 0$			
			$\frac{(\beta)_t (\gamma)_t}{(t-n)! (\delta)_t}$						

\* - uniform grid



Table 3. Orthogonal bases of a continuous argument

No.	Orthogonal basis	Symbol	General expression	Weight $\rho(t)$	Limits of existence	
					Lower	Upper
1	Shifted Jacobi polynomials	$\beta > -1$ $\mathfrak{P}_n^{\alpha\beta}(t, T)$	$\sqrt{\frac{(\alpha + \beta + 2n + 1)\Gamma(\alpha + \beta + n + 1)\Gamma(\alpha + n + 1)\Gamma(\beta + n + 1)}{2^{\alpha + \beta} T n!}} \frac{1}{T^n} \sum_{k=0}^n \frac{C_n^k(t-T) t^k n^{-k}}{\Gamma(\alpha + k + 1)\Gamma(\beta + n - k + 1)}$	$\frac{2^{\alpha + \beta}}{T^{\alpha + \beta}} (T-t)^\alpha t^\beta$	0	T
		$\alpha > -1$ $R_n^{\alpha\beta}(mT)$	$\sqrt{\frac{m(\alpha + \beta + 2n + 1)\Gamma(\alpha + \beta + n + 1)\Gamma(\alpha + n + 1)\Gamma(\beta + n + 1)}{2^{\alpha + \beta} n!}} \sum_{k=0}^n \frac{(-1)^k C_n^k e^{-(0.5+k)mt} (1 - e^{-mt})^{n-k}}{\Gamma(\alpha + k + 1)\Gamma(\beta + n - k + 1)}$	$2^{\alpha + \beta} e^{-\alpha mt} (1 - e^{-mt})^\beta$	0	$\infty$
2	Shifted Jacobi functions	$\beta > -1$ $\mathfrak{P}_n^{\alpha\beta}(t, T)$	$\sqrt{\frac{(\alpha + \beta + 2n + 1)\Gamma(\alpha + \beta + n + 1)\Gamma(\alpha + n + 1)\Gamma(\beta + n + 1)}{n! T^{2n + \alpha + \beta + 1}}} \sum_{k=0}^n \frac{(-1)^k C_n^k}{\Gamma(\alpha + k + 1)} (T-t)^{\frac{\alpha}{2} + k} t^{\frac{\beta}{2} + n - k}$	1	0	T
		$\alpha > -1$ $r_n^{\alpha\beta}(mt)$	$\sqrt{\frac{m(\alpha + \beta + 2n + 1)\Gamma(\alpha + \beta + n + 1)\Gamma(\beta + n + 1)}{n!}} \sum_{k=0}^n \frac{(-1)^k C_n^k}{\Gamma(\alpha + k + 1)\Gamma(\beta + n - k + 1)} e^{-\frac{\alpha + 1 + 2k}{2} mt} (1 - e^{-mt})^{n-k}$	1	0	$\infty$
3	Shifted Gegenbauer polynomials	$\beta = \sigma - 0.5$ $C^\sigma(t, T)$	$\sqrt{\frac{(\sigma + n)\Gamma(2\sigma + n)}{2^{2\sigma - 2} T n!}} \Gamma(\sigma + n + 0.5) \frac{1}{T^n} \sum_{k=0}^n \frac{C_n^k(t-T) t^k n^{-k}}{\Gamma(\sigma + k + 0.5)\Gamma(\sigma + n - k + 0.5)}$	$\frac{2^{2\sigma - 1}}{T^{2\sigma - 1}} (T-t)^{\sigma - 0.5} t^{\sigma - 0.5}$	0	T
		$\alpha - \beta = \sigma - 0.5$ $C^\sigma(mt)$	$\sqrt{\frac{m(\sigma + n)\Gamma(2\sigma + n)}{2^{2\sigma - 2} n!}} \Gamma(\sigma + n + 0.5) \sum_{k=0}^n \frac{(-1)^k C_n^k e^{-(0.5+k)mt}}{\Gamma(\sigma + k + 0.5)\Gamma(\sigma + n - k + 0.5)} (1 - e^{-mt})^{n-k}$	$2^{2\sigma - 1} e^{-(\sigma - 0.5)mt} \times (1 - e^{-mt})^{\sigma - 0.5}$	0	$\infty$

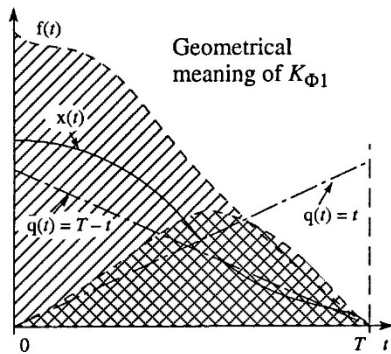


$$x(t) = \sum_{n=0} A_n \varphi_n(t)$$

$\varphi_n(t)$  - functional orthogonal basis



### Form coefficient

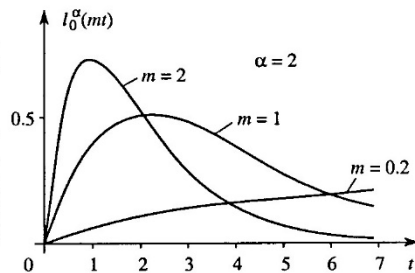


$$K_{\Phi q} = \frac{\bar{J}_q}{J_q} = \frac{\int_0^T x(t) \bar{q}(t) dt}{\int_0^T x(t) q(t) dt}; \quad K_{\Phi q} = \frac{\sum_{t=0}^N x(t) \bar{q}(t)}{\sum_{t=0}^N x(t) q(t)}$$

$t$  – continuous argument,  $t$  – discrete argument.

$x(t)$  – the signal

$q(t)$  – known function



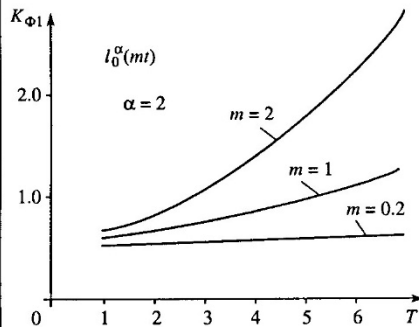
For  $q(t) = t^n, \bar{q}(t) = (T-t)^n$

For  $q(t) = e^{-at}, \bar{q}(t) = e^{-a(T-t)}$

$$K_{\Phi n} = \frac{\int_0^T x(t) (T-t)^n dt}{\int_0^T x(t) t^n dt}, \quad K_{\Phi e} = \frac{\int_0^T x(t) e^{-a(T-t)} dt}{\int_0^T x(t) e^{-at} dt}$$

power form coefficient

exponential form coefficient

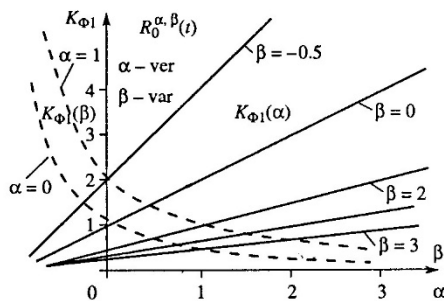


$$K_{\Phi 1} = \frac{\int_0^T x(t) (T-t) dt}{\int_0^T x(t) t dt} = \frac{T \int_0^T x(t) dt - \int_0^T x(t) t dt}{\int_0^T x(t) t dt} = T \frac{J_0}{J_1} - 1.$$

If  $K_{\Phi 1} > 1$ , then  $x(t)$  – decays on  $[0, T]$ ;

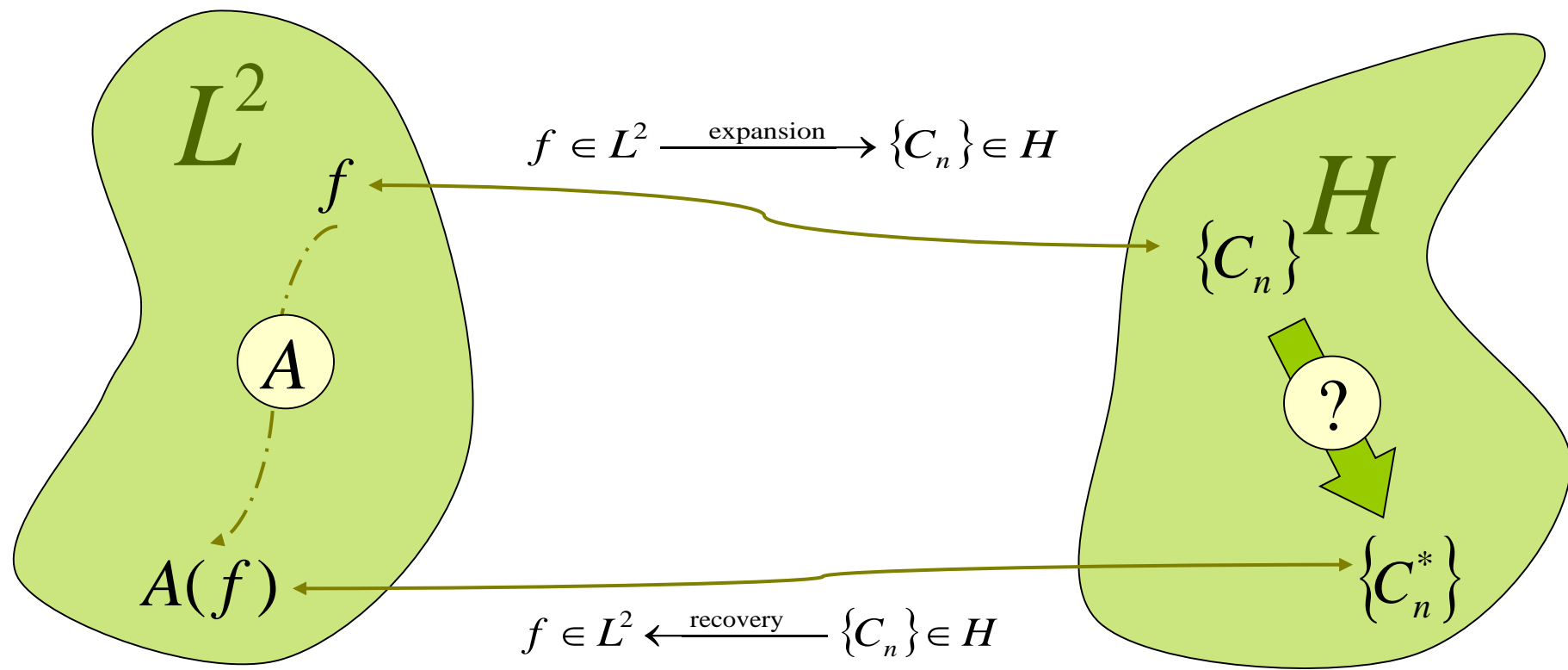
If  $K_{\Phi 1} < 1$ , then  $x(t)$  – increases on  $[0, T]$ ;

If  $K_{\Phi 1} \approx |1|$ , then  $x(t)$  – is periodical (nondecreasing).





# Transformations in the space of coefficients



Long-term numerical computation

The acceleration

$$f(x) \xrightarrow{\text{Digital transformations } A()} A(f(x))$$

$$\{C_n\} \xrightarrow{\text{How to calculate?}} \{C_n^*\}_9$$

$$\hat{L} = \frac{d^k}{dx^k} + a_1 \frac{d^{k-1}}{dx^{k-1}} + \dots + a_{k-1} \frac{d}{dx} + a_k$$

$$\hat{L}y = f$$

$$\hat{g}y \Big|_s = f_0$$

$$\hat{g} = \frac{d^{k-1}}{dx^{k-1}} + b_1 \frac{d^{k-2}}{dx^{k-2}} + \dots + b_{k-2} \frac{d}{dx} + b_{k-1}.$$

$$y = \sum_{i=0}^N p_i T_i, \quad f = \sum_{i=0}^N h_i T_i \quad \text{и} \quad \sum_{i=0}^N a_i \hat{g} T_i \Big|_s = f_0$$

# Derivative approximation

$$y'_N(t) = \sum_{i=0}^N d_i^{(1)} T_i(t),$$

$$\mathbf{d}^{(1)} = \mathbf{D}_N \mathbf{p},$$

$$\mathbf{d}^{(\mathbf{k})} = \mathbf{D}_{N-\mathbf{k}+1} \cdot \dots \cdot \mathbf{D}_N \mathbf{p}.$$

$$\hat{L} \left\{ d_{ij}^{(\hat{L})} \right\} = \left\{ d_{ij}^{(k)} + a_1 d_{ij}^{(k-1)} + \dots + a_{k-1} d_{ij}^{(1)} + a_k \right\}$$

Derivative approximation in Chebyshev basis

$$\mathbf{D}_N = \overbrace{N-1} \left( \begin{array}{c|cccccc} & \sqrt{2} & 0 & 3\sqrt{2} & 0 & 5\sqrt{2} & \dots \\ & 0 & 4 & 0 & 8 & 0 & \dots \\ 0 & 0 & 0 & 6 & 0 & 10 & \dots \\ & & & \ddots & & & \\ & & & & \ddots & & \\ & 0 & 0 & \dots & 0 & 0 & 2N \end{array} \right)$$

$$b_0 = \sqrt{2}p_1 + 3\sqrt{2}p_3 + 5\sqrt{2}p_5 + \dots + N\sqrt{2}p_N$$

$$b_1 = 4p_2 + 8p_4 + 12p_6 + \dots + 2(N-1)p_{N-1}$$

$$b_2 = 6p_3 + 10p_5 + 14p_7 + \dots + 2Np_N$$

.....

$$b_{N-1} = 2Np_N$$

Derivative approximation in Legendre basis

$$\mathbf{D}_N = N-1 \left\{ \begin{array}{c} \overbrace{\left( \begin{array}{cccccc} \sqrt{3} & 0 & \sqrt{7} & 0 & \sqrt{11} & \dots \\ 0 & \sqrt{3}\sqrt{5} & 0 & \sqrt{3}\sqrt{9} & 0 & \dots \\ 0 & 0 & \sqrt{5}\sqrt{7} & 0 & \sqrt{5}\sqrt{11} & \dots \\ & & \ddots & & & \\ & & & \ddots & & \\ 0 & 0 & \dots & 0 & 0 & \sqrt{2N-1}\sqrt{2N+1} \end{array} \right)}^N \end{array} \right.$$

# Example

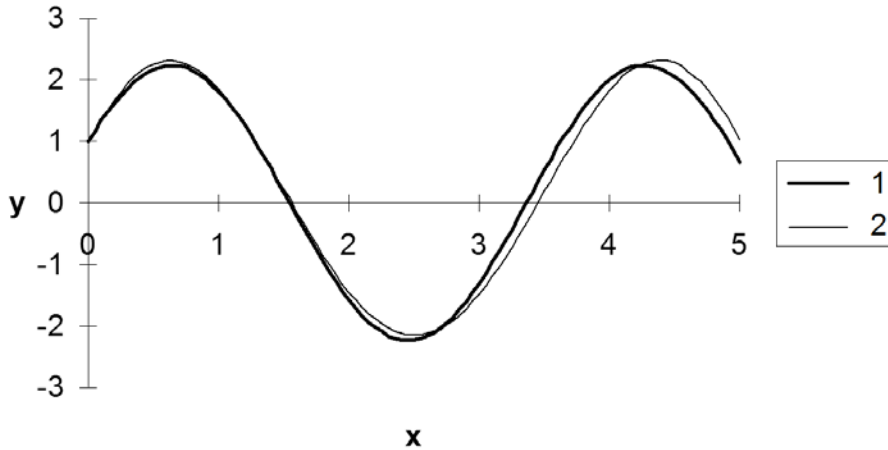
$$u'' + 3u = 0,$$

$$u(0) = 1,$$

$$u'(5) = 2\sqrt{3} \cos(5\sqrt{3}) - \sqrt{3} \sin(5\sqrt{3}).$$

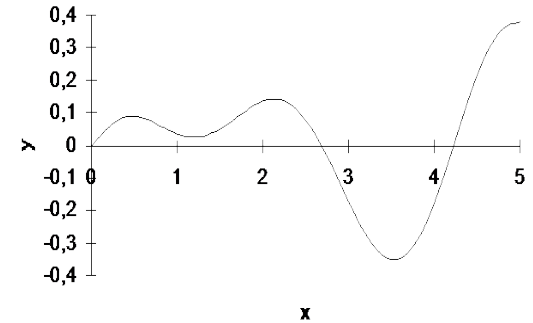
$$u(x) = \cos(\sqrt{3}x) + \sin(\sqrt{3}x).$$

N=8

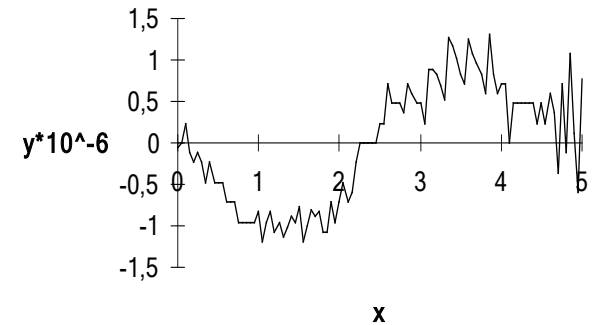


$$u(x) = \sum_{k=0}^{N=8,16} A_k \varphi_k(x)$$

N=8



N=16



$$f(x) = \int y(x) dx$$

$$\mathbf{b} = \mathbf{I}_n \mathbf{h}$$

$$\mathbf{b} = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \dots \\ \beta_N \end{pmatrix}$$

$$\mathbf{h} = \begin{pmatrix} \eta_1 \\ \eta_2 \\ \dots \\ \eta_N \end{pmatrix}$$

$$\mathbf{I}_N = N \begin{pmatrix} \overbrace{\hspace{10em}}^N \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{2} & 0 & 0 & \dots \\ 0 & \frac{1}{4} & 0 & -\frac{1}{4} & 0 & \dots \\ 0 & 0 & \frac{1}{6} & 0 & -\frac{1}{6} & \dots \\ & & \ddots & & & \\ & & & \ddots & & \\ 0 & 0 & \dots & 0 & 0 & \frac{2}{2(N+1)} \end{pmatrix}$$

$$\beta_1 = \frac{\eta_0}{\sqrt{2}} - \frac{\eta_2}{2},$$

$$\beta_2 = \frac{\eta_1}{4} - \frac{\eta_3}{4},$$

$$\beta_3 = \frac{\eta_2}{6} - \frac{\eta_4}{6},$$

.....

$$b_{N+1} = \frac{1}{2N+1} \eta_N$$



# Wiener-Hopf equation

$$u(x) - \int_0^{\infty} R(x-s)u(s)ds = f(x)$$

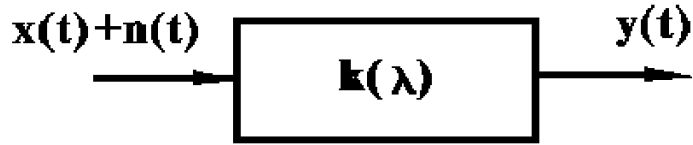
$$R(x) = R_1(x) + \delta(x),$$

$$f_1(x) = -f(x)$$

$$\int_{-\infty}^{+\infty} \varphi(s) \delta(x-s) ds = \varphi(x)$$

$$\int_0^{\infty} R_1(x-s)u(s)ds = f_1(x)$$

$$R_{xy}(\tau) = \int_0^{\infty} R_{xx}(\tau - \lambda)k(\lambda)d\lambda,$$



$$y(t) = \int_0^t x(t - \tau)k(\tau)d\tau.$$

$$\begin{aligned} R_{xy}(\tau) &= \int_0^{\infty} R_{xx}(\tau)k(\lambda)d\lambda = -\int_{\tau}^{\infty} R_{xx}(\tau)k(\tau - \lambda)d\lambda = \\ &= \int_{-\infty}^0 R_{xx}(\tau)k(\tau - \lambda)d\lambda + \int_0^{\tau} R_{xx}(\lambda)k(\tau - \lambda)d\lambda. \end{aligned}$$

$$\begin{aligned} \int_{-\infty}^0 R_{xx}(\lambda)k(\tau - \lambda)d\lambda &= \int_{\infty}^0 R_{xx}(-\lambda)k(\tau + \lambda)d(-\lambda) = \\ &= \int_0^{\infty} R_{xx}(-\lambda)k(\tau + \lambda)d\lambda = \int_0^{\infty} R_{xx}(\lambda)k(\tau + \lambda)d\lambda. \end{aligned}$$

$$R_{xx}(\tau) = \sum_{i=0}^N C_i l_i(m\tau),$$

$$R_{xy}(\tau) = \sum_{i=0}^N D_i l_i(m\tau).$$

$$k(\lambda) = \sum_{i=0}^N E_i l_i(m\tau)$$

$l_i$  -  $i^{th}$  Laguerre function with parameter  $m > 0$

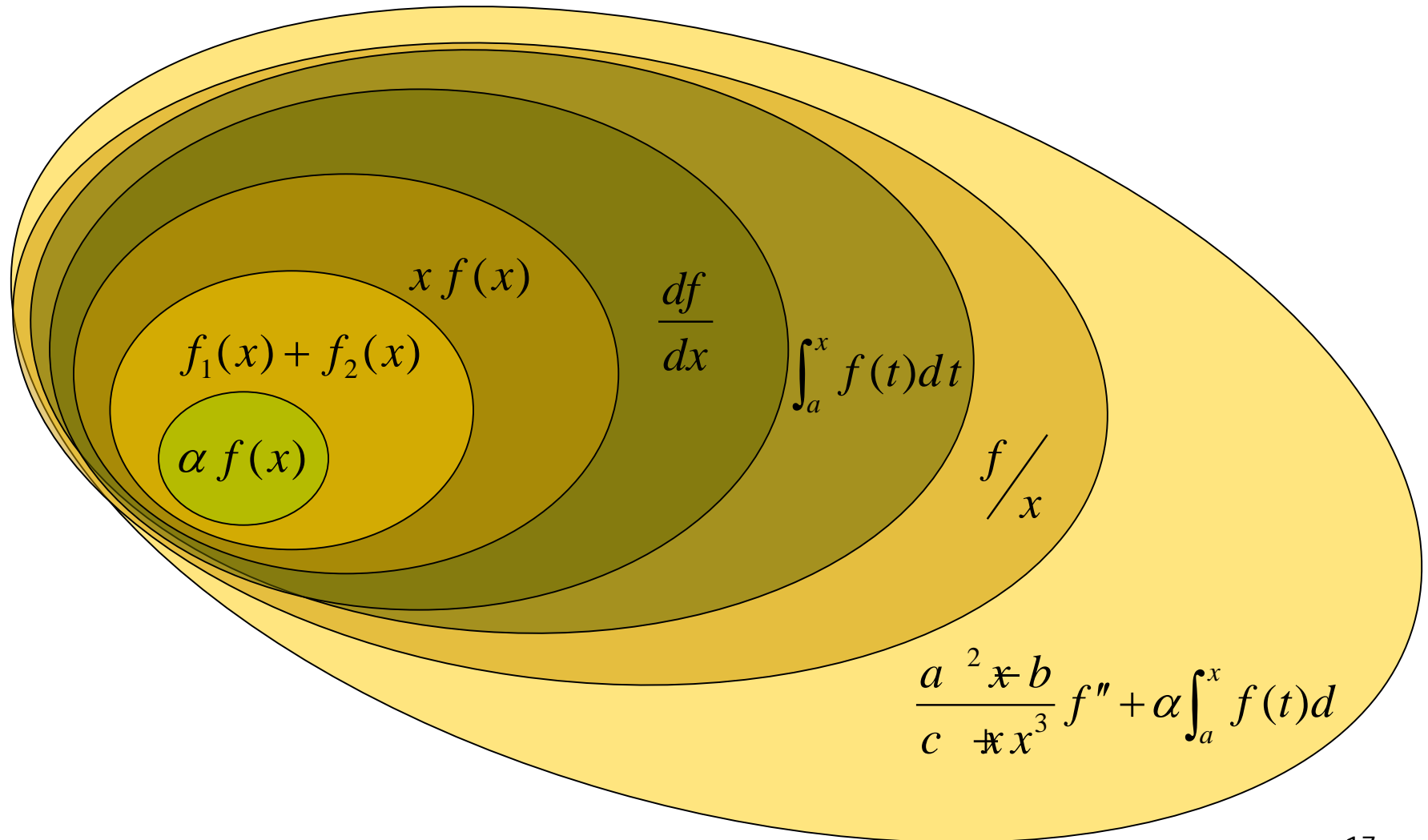
$$\int_0^{\infty} R_{xx}(\lambda)k(\tau + \lambda)d\lambda = \sum_{\rho=0}^N C_{\rho} \sum_{j=0}^N E_j \int_0^{\infty} l_{\rho}(m\lambda)l_j(m\tau + m\lambda)d\lambda.$$

$$l_n(m(x + y)) = \frac{1}{\sqrt{m}} \sum_{j=0}^n l_j(mx) \{l_{n-j}(my) - l_{n-j-1}(my)\}$$

$$\sqrt{m}D_i = C_0 E_i + \sum_{j=0}^N E_j (C_{|i-j|} - C_{|i-j|-1}),$$

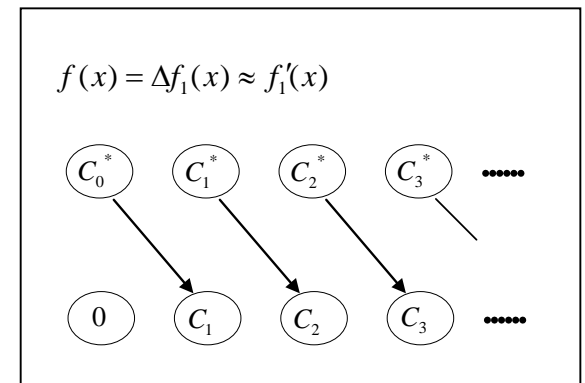
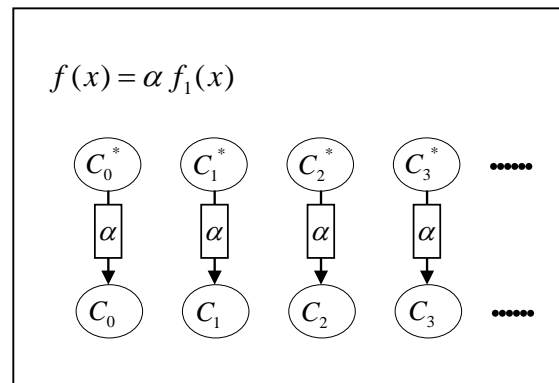
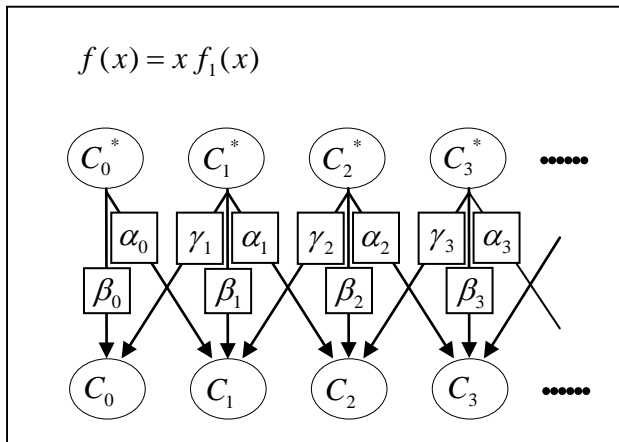
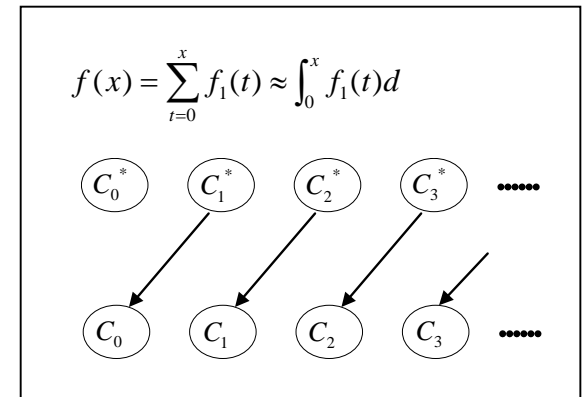
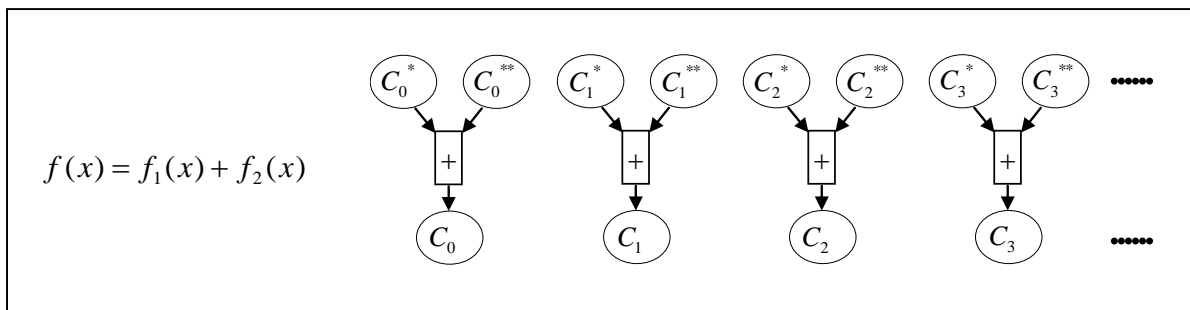
$$C_i \equiv 0 \text{ for } \forall i < 0,$$

# Operators, realized as a quick 'autospectral' algorithms



# Recurrent calculations of coefficients. The method of fast series transformation $O(N)$

Analytical transformations and the corresponding changes in the spectral coefficients using orthogonal polynomials of Kravchuk



# Parallel computing for the ultrafast transformation of spectral series $O(\ln N)$

Functional calculation

$$f = \int (\alpha f_1' + x^2 f_2) dx$$

$$f(x) \approx \sum_{t=0}^x (\alpha \Delta f_1(t) + x^2 f_2(t))$$

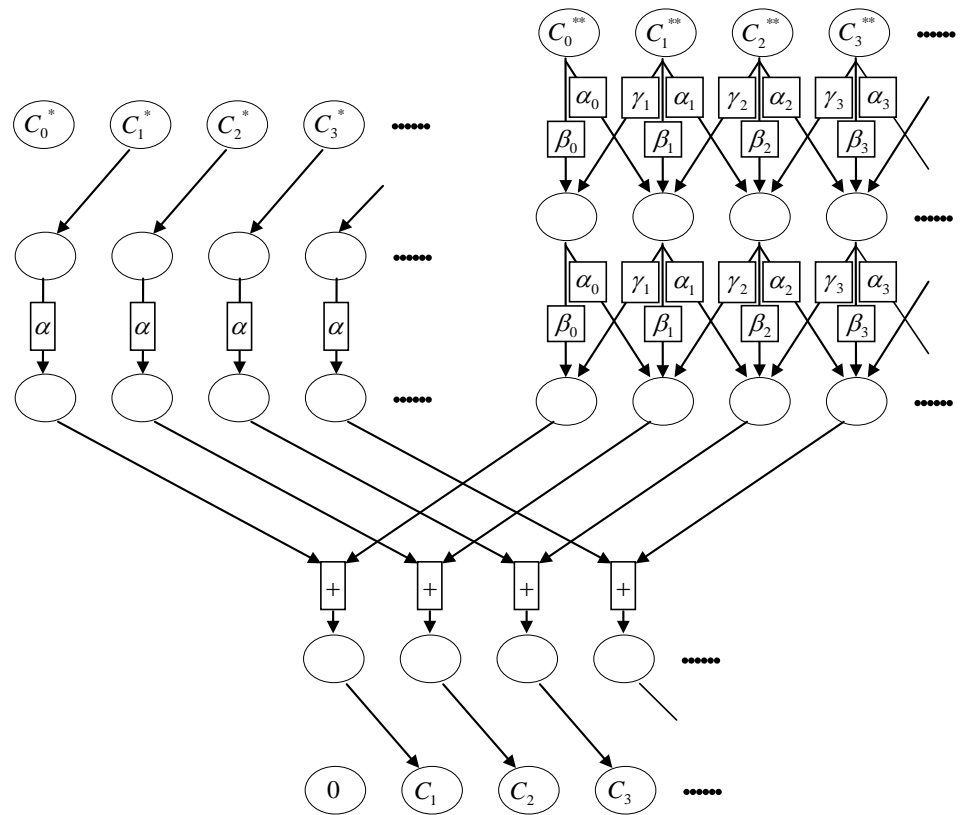
By means of expansion coefficients

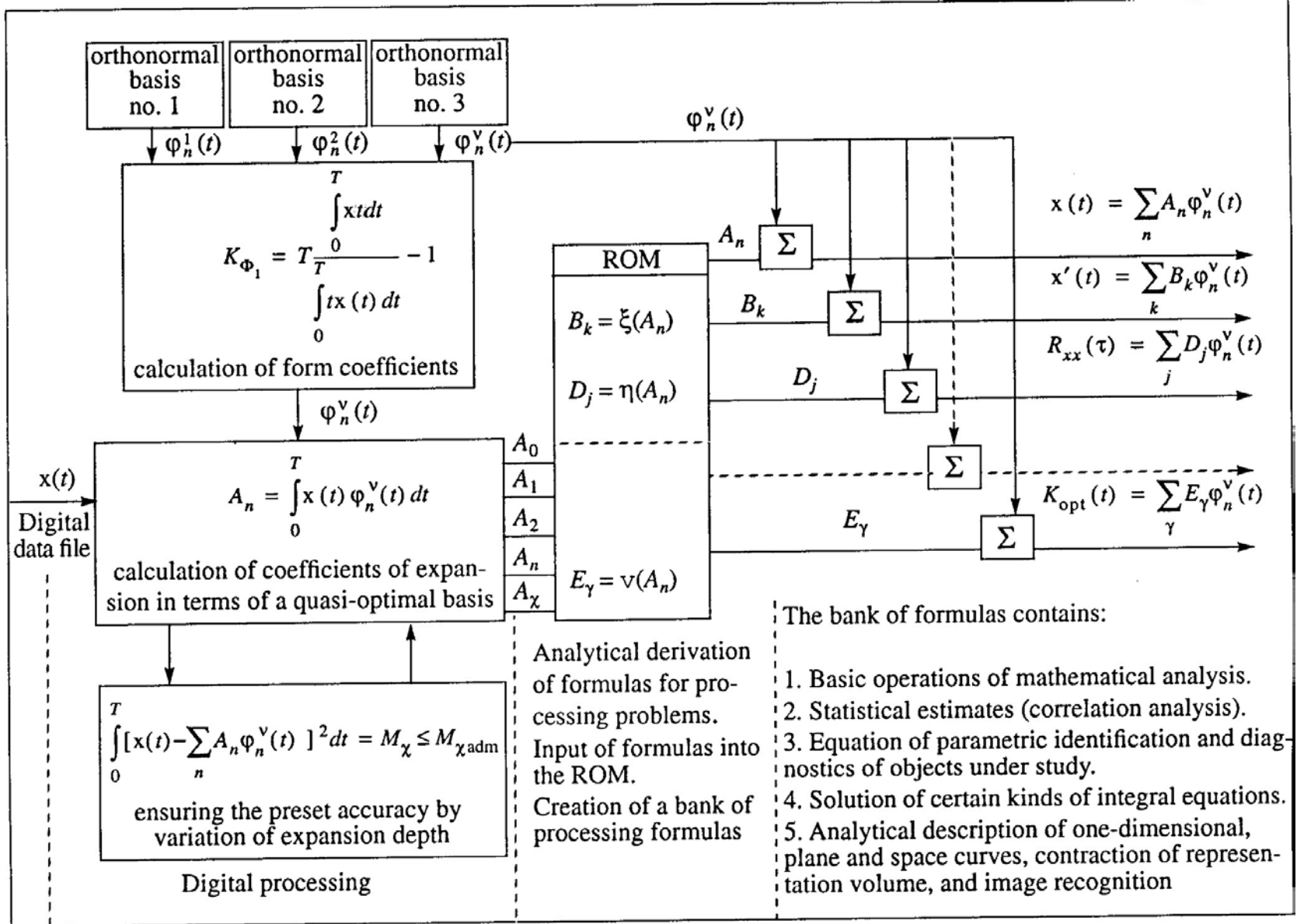
Spectra of  $f_1$  and  $f_2$ :

and  $\{C_0^*, C_1^*, C_2^*, \dots\}$   
 $\{C_0^{**}, C_1^{**}, C_2^{**}, \dots\}$ .

Result – spectrum of  $f(x)$ :

$\{C_0, C_1, C_2, \dots\}$





$$x(t) \approx \sum_{n=0}^{N=N_{\min}} A_n \varphi_n(t)$$



# IMAGE ANALYSIS AND PATTERN RECOGNITION

Parametric representation of space curves

$$\left. \begin{aligned} x &= x(t) \\ y &= y(t) \\ z &= z(t) \end{aligned} \right\} \text{The vector form}$$

$$\mathbf{F} = \mathbf{i}x(t) + \mathbf{j}y(t) + \mathbf{k}z(t)$$

$$x(t) = \sum_{n=0}^N A_n \varphi_n(t);$$

$$y(t) = \sum_{k=0}^K B_k \psi_k(t);$$

$$z(t) = \sum_{i=0}^J C_i \eta_i(t);$$

The root-mean-square error of the approximation:

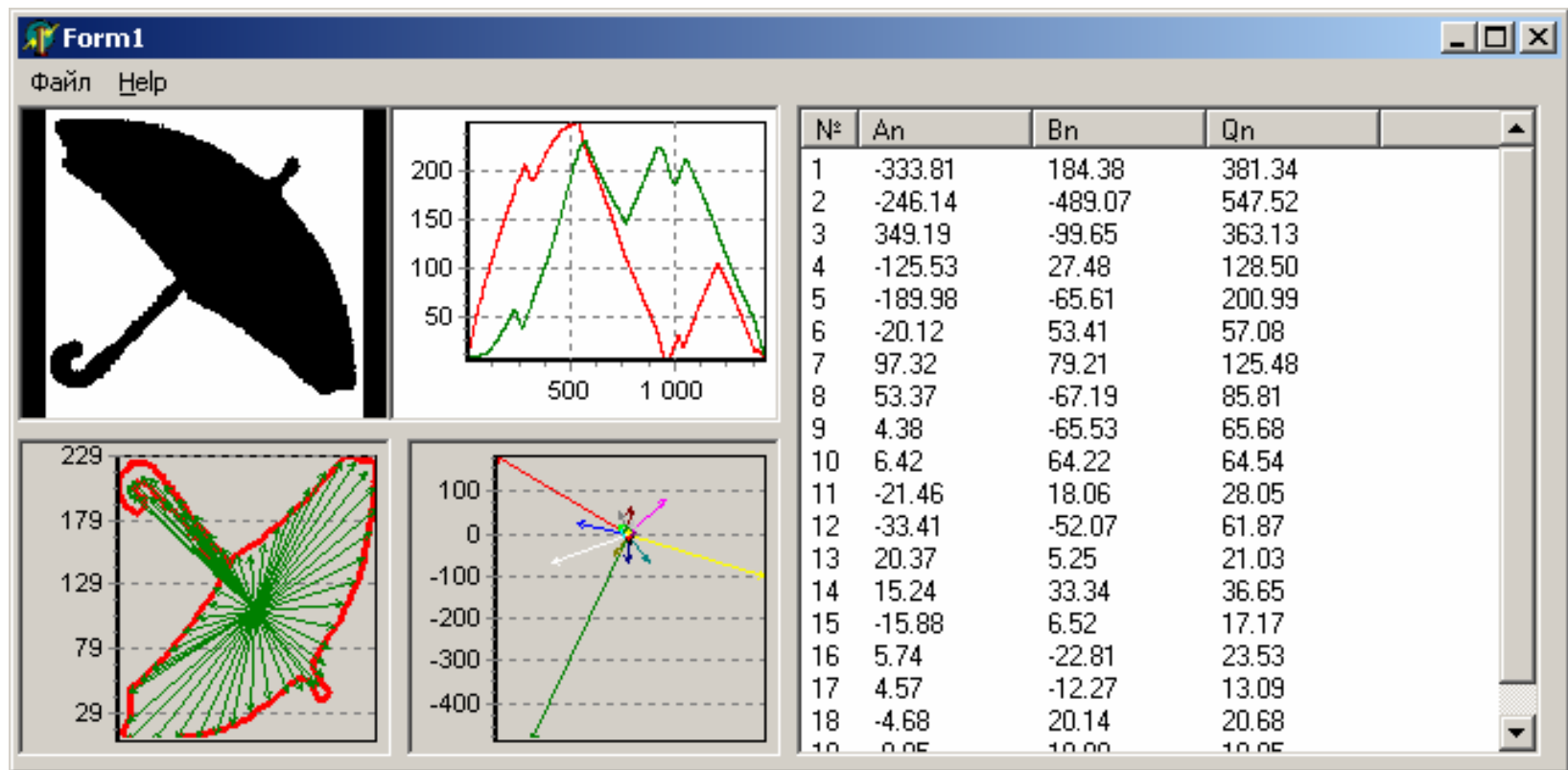
$$\theta_n = \frac{\int_a^b \left[ x(t) - \sum_{n=0}^N A_n \varphi_n(t) \right]^2 dt}{\int_a^b x^2(t) dt}$$

$$= 1 - \frac{\sum_{n=0}^N A_n^2}{\int_a^b x^2(t) dt}$$

or accuracy:

$$\gamma_n = 1 - \theta_n = \frac{\sum_{n=0}^N A_n^2}{\int_a^b x^2(t) dt} = \frac{\sum_{n=0}^N A_n^2}{B_x}$$

# Analytical description of contours and selection of analytical features



# Software for OCR

Form1

File Edit Options About

LOAD IMAGE hello REFRESH

Threshold

Lite  Strong  Custom

Middle  Heavy 200

Preview

Auto refresh hello

Restrictions

Significant figures size 40

Significant holes size 12

Significant diacritics size 12

Smooth shapes

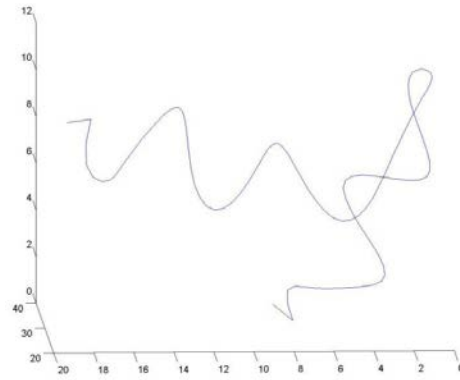
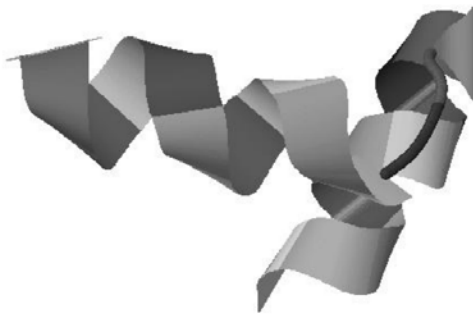
Alphabet	Frequency
a	i re
b	j s
c	k st
d	l t
e	m u
ed	n v
eo	o w
f	p x
g	q y
h	r z

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Cumulativ Abscis Ordina Radial Azimut Thick Coher Holes Diactr Collin Lean SizeUp

1 1 10 10 10 10 5 1 5 1 1

TetJev RQ www.math46s.com



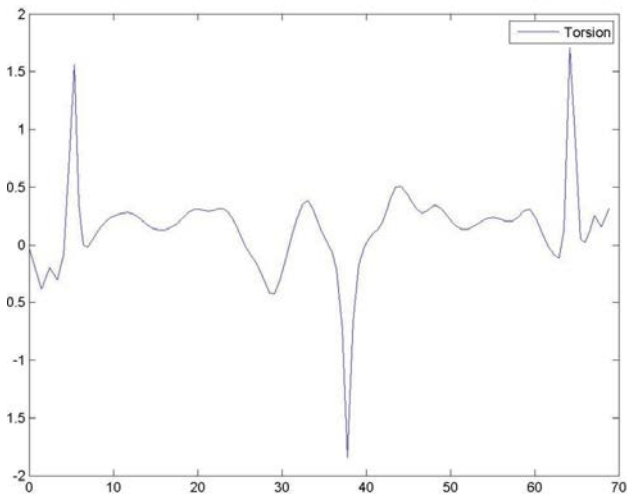
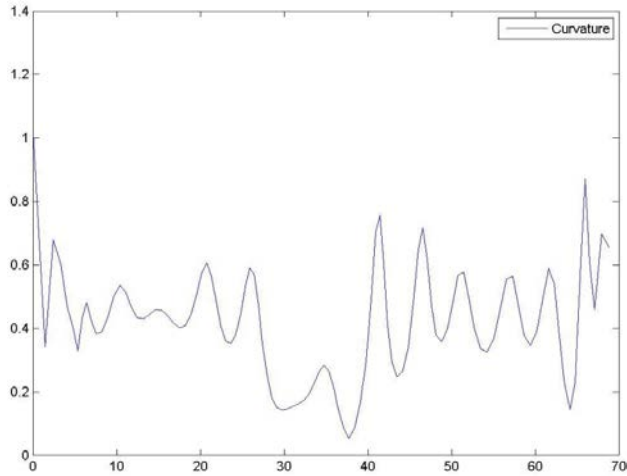
$$\begin{cases} x = x(t), \\ y = y(t), \\ z = z(t) \end{cases}$$



$$\begin{cases} x(t) = \sum_{i=0}^N A_i \varphi_i(t) \\ y(t) = \sum_{i=0}^N B_i \varphi_i(t) \\ z(t) = \sum_{i=0}^N C_i \varphi_i(t) \end{cases}$$



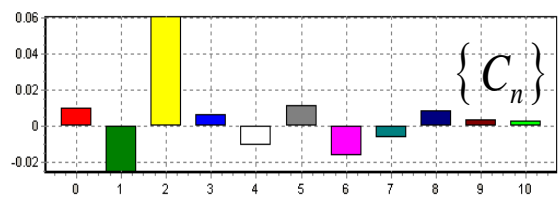
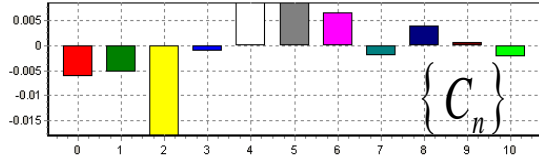
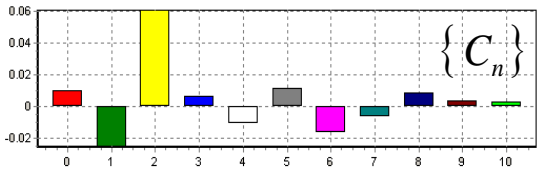
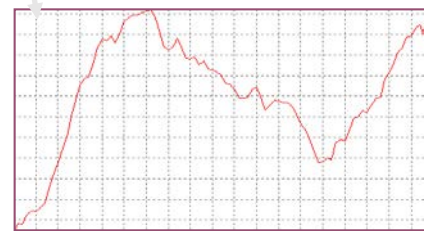
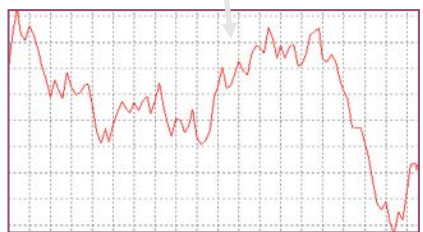
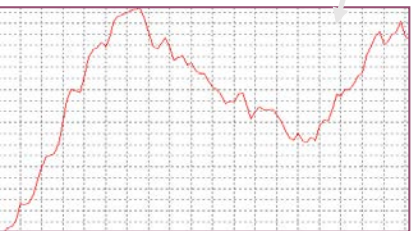
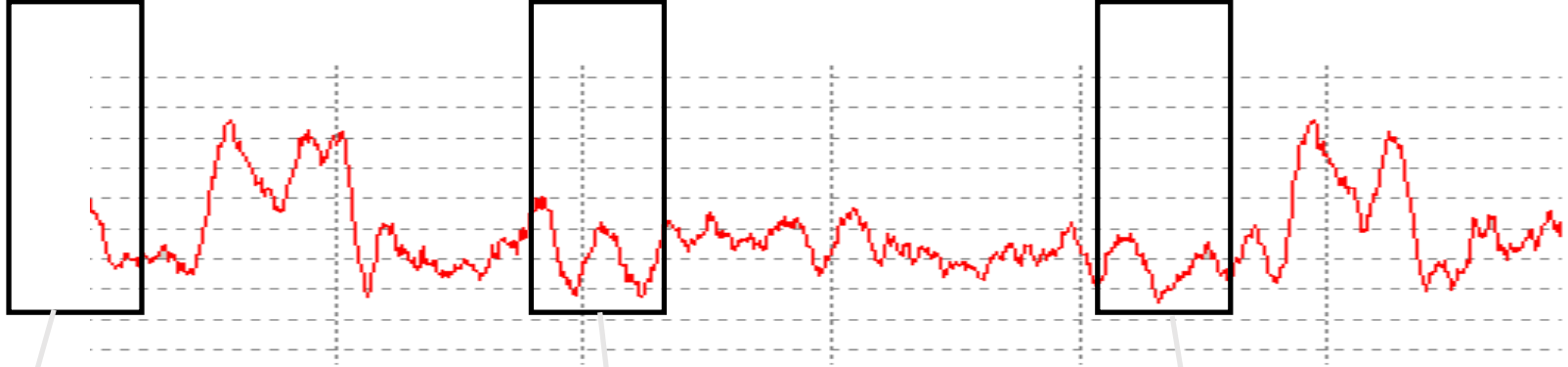
$$\begin{cases} C(s) = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}, \\ T(s) = \frac{\begin{vmatrix} \dot{x} & \dot{y} & \dot{z} \\ \ddot{x} & \ddot{y} & \ddot{z} \\ \ddot{x} & \ddot{y} & \ddot{z} \end{vmatrix}}{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} \end{cases}$$



# Repeats search in genomes

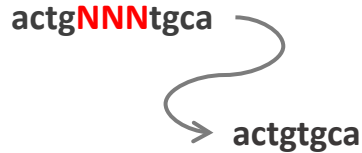
ATGXGXATTXTXTGXXTGXATAAATXGXHXGTATAAAXHXGTAX

$f_i$

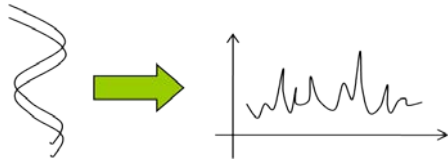


# General scheme of the algorithm

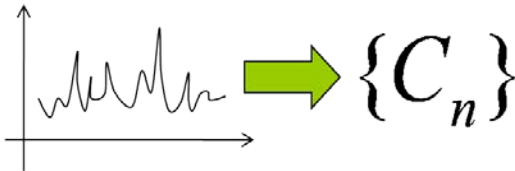
actgNNNtgca  
actgtgca



Preliminary DNA processing  
последовательностей



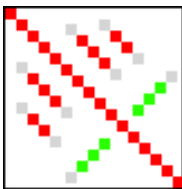
Preparation of DNA  
profiles : GC%, GA%



Converting the DNA profiles  
in the spectral representation

$$\theta(\{C_n\}, \{C'_n\}) < \varepsilon$$

Comparison of the  
spectra  
of DNA fragments profiles

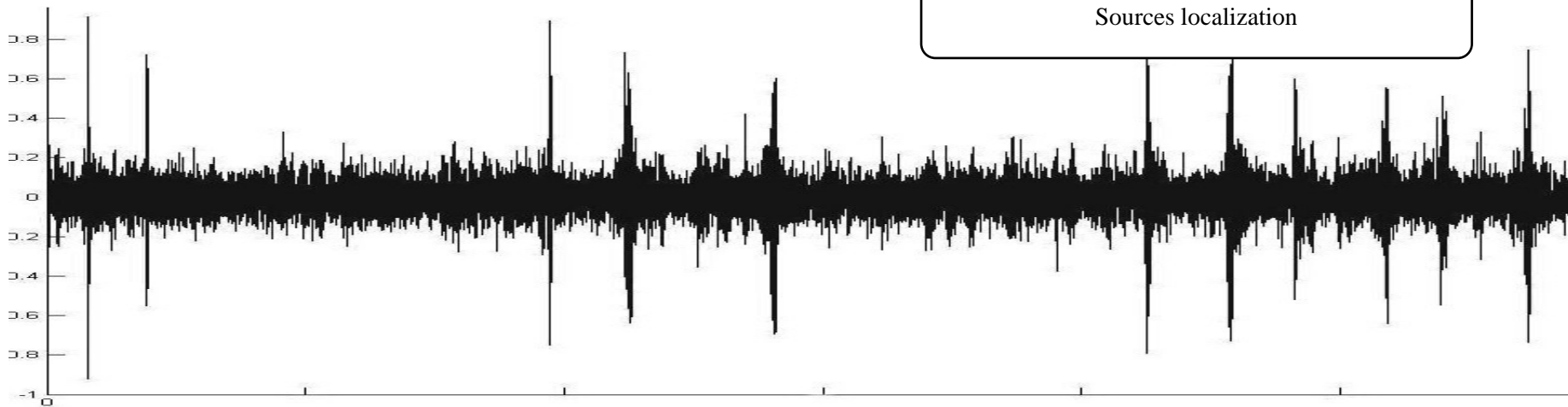
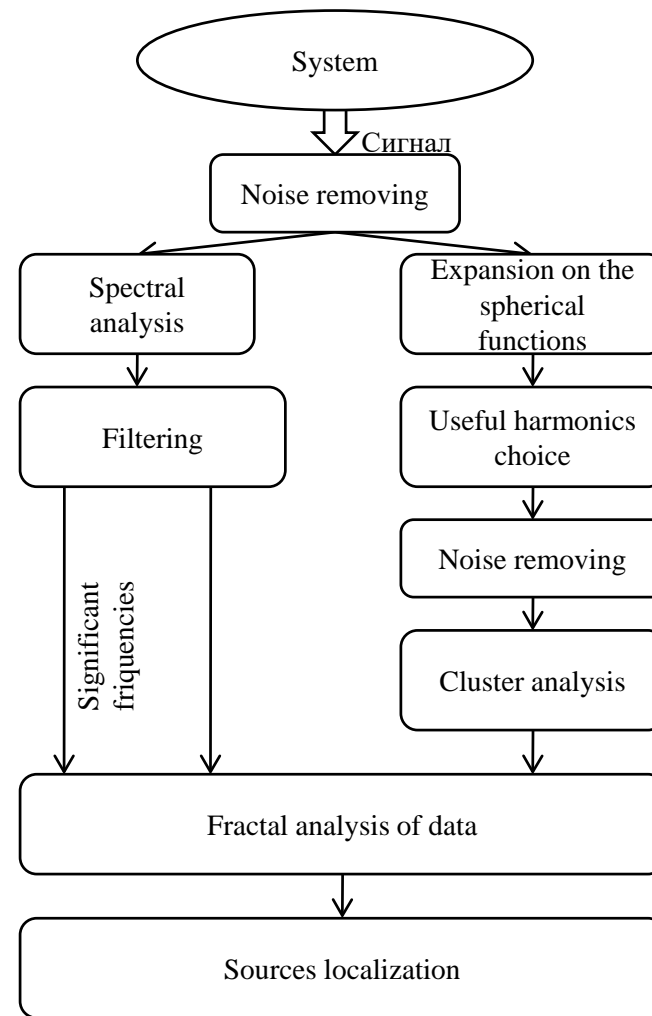
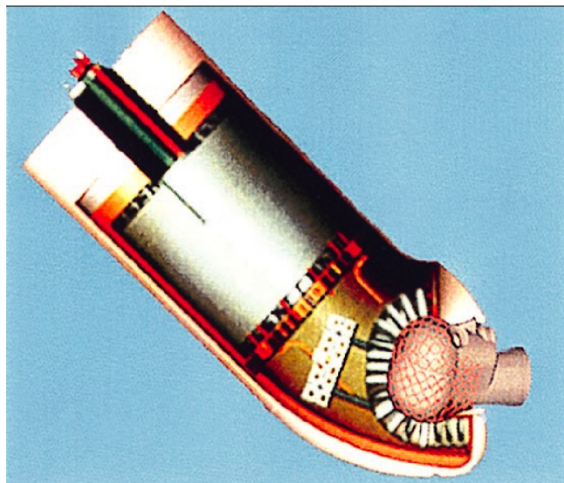


Showing results in the matrix of  
spectral similarity and analysis

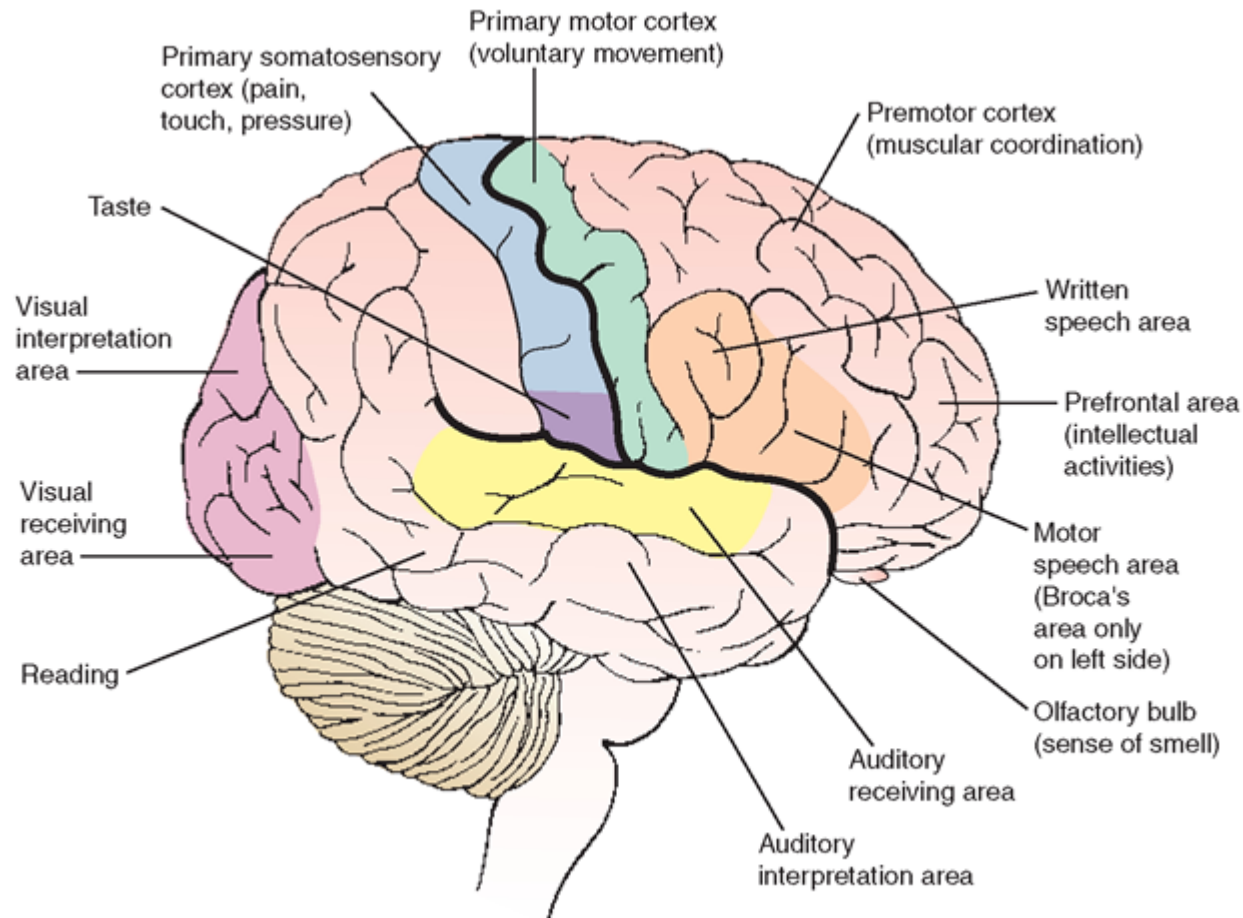


# Magnetic encephalography

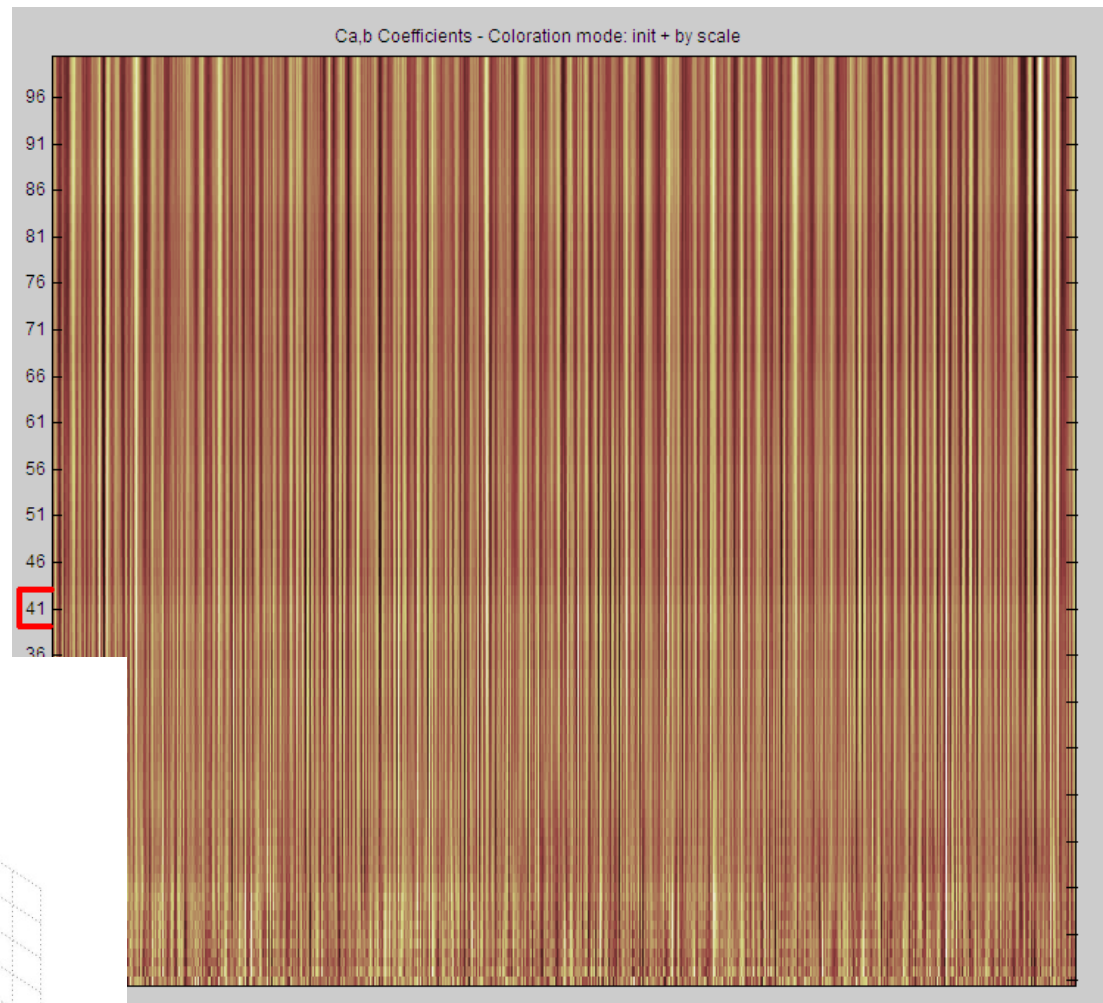
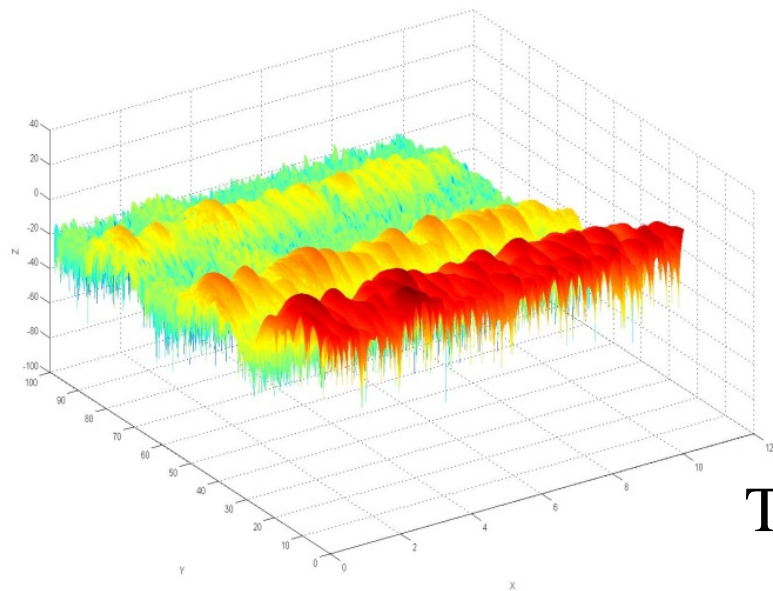
Measuring complex Magnes 2500 WH  
(New York, USA)



# The brain functional areas mapping



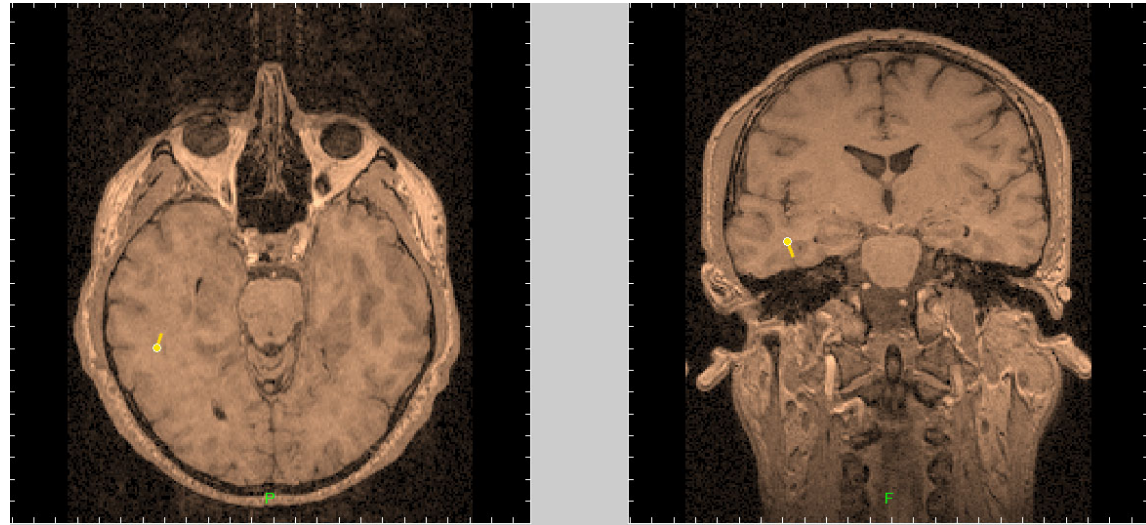
# Audio stimulation



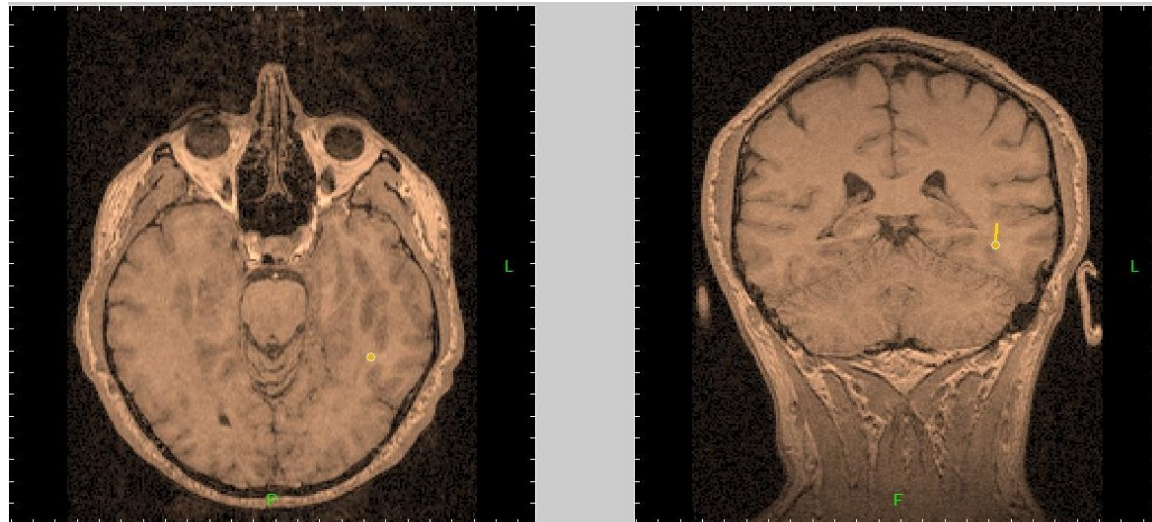
The wavelet coefficients in the Haar basis

# Localization of the source feeding the audio stimulus

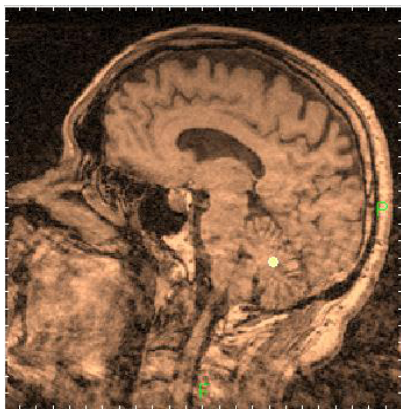
localization of the  
magnetic field  
source at 10 Hz



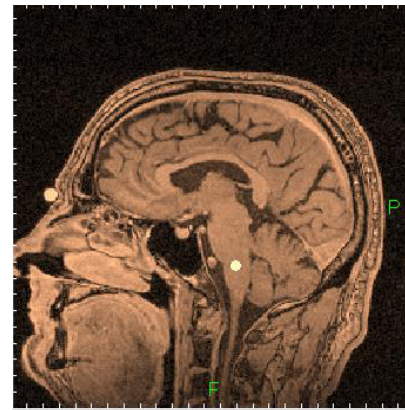
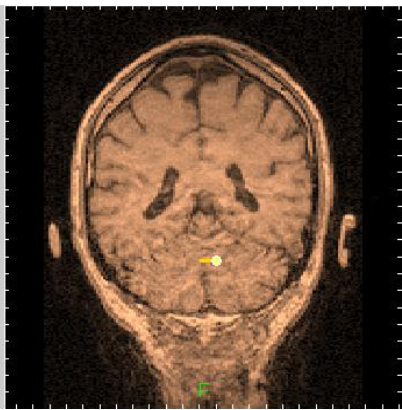
20 Hz



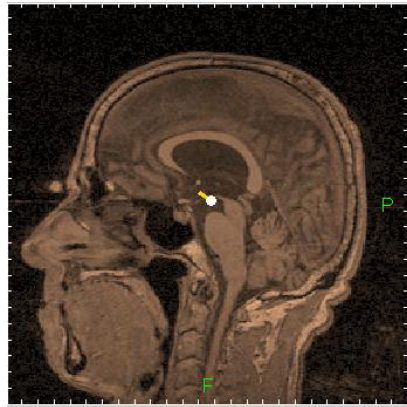
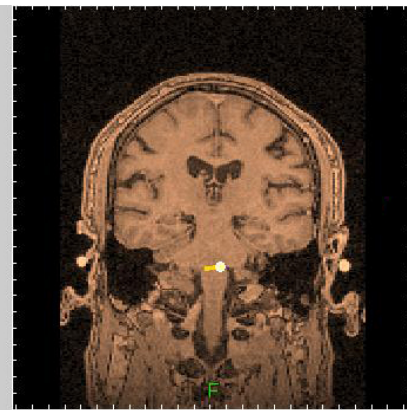




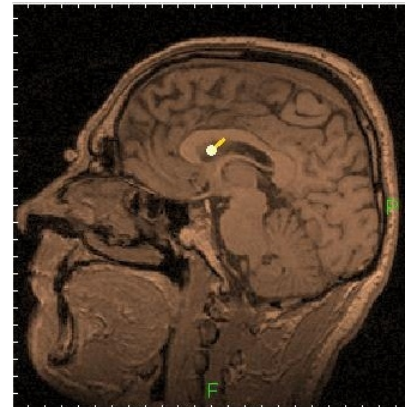
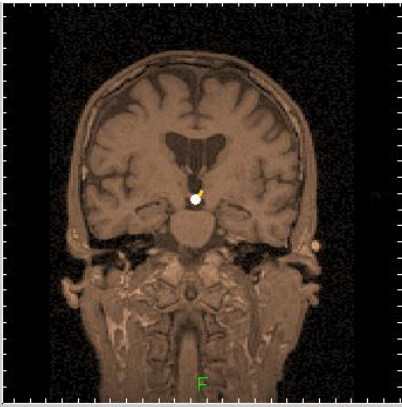
*a source in the cerebellum*



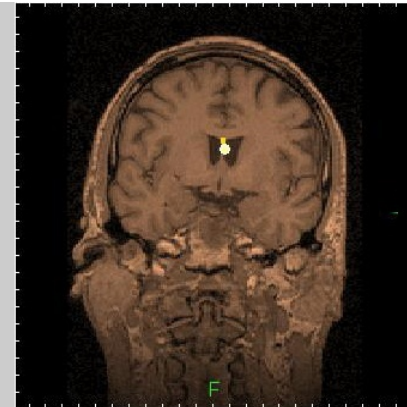
*... in the brainstem (pons)*



*... in substantia nigra*



*... in the caudate nucleus*



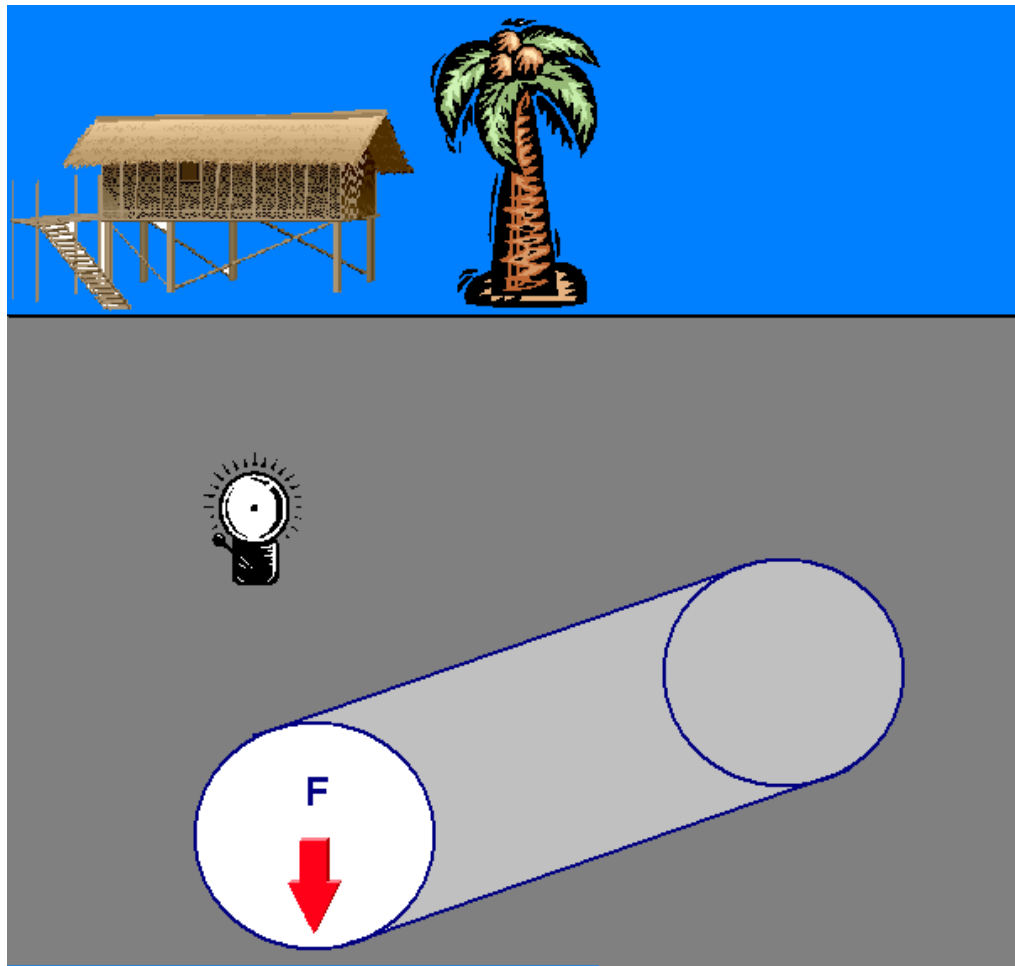
## Parkinsonism

# Vibro-acoustic ecology of the city





# The problem of vibro-acoustic control



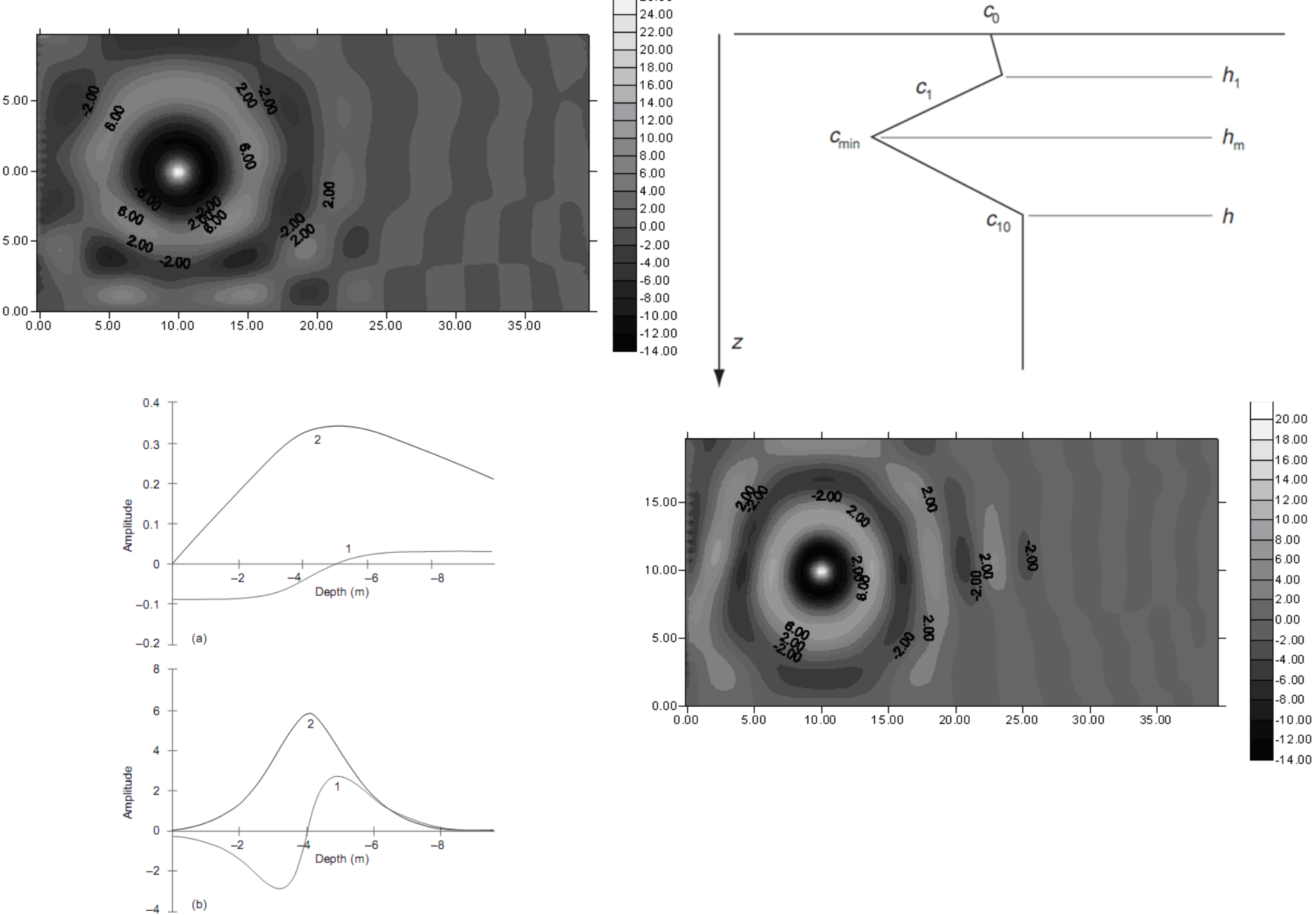
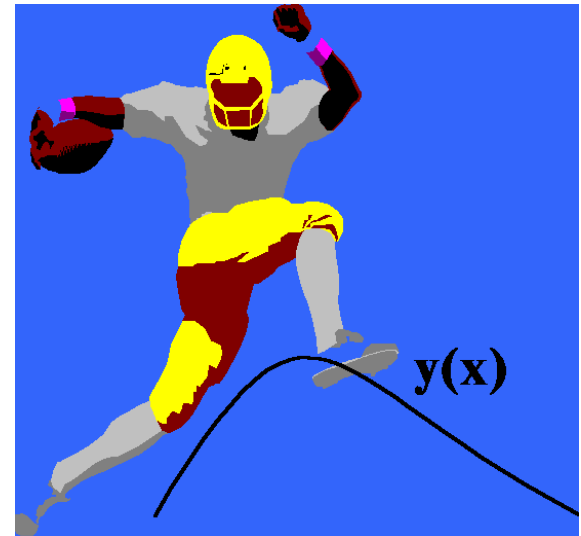


Fig. 13.5. Dependence of the vibration acceleration (curve 1) and pressure (curve 2) on depth (in relative units): (a)  $f = 31.5$  Hz; (b)  $f = 125$  Hz

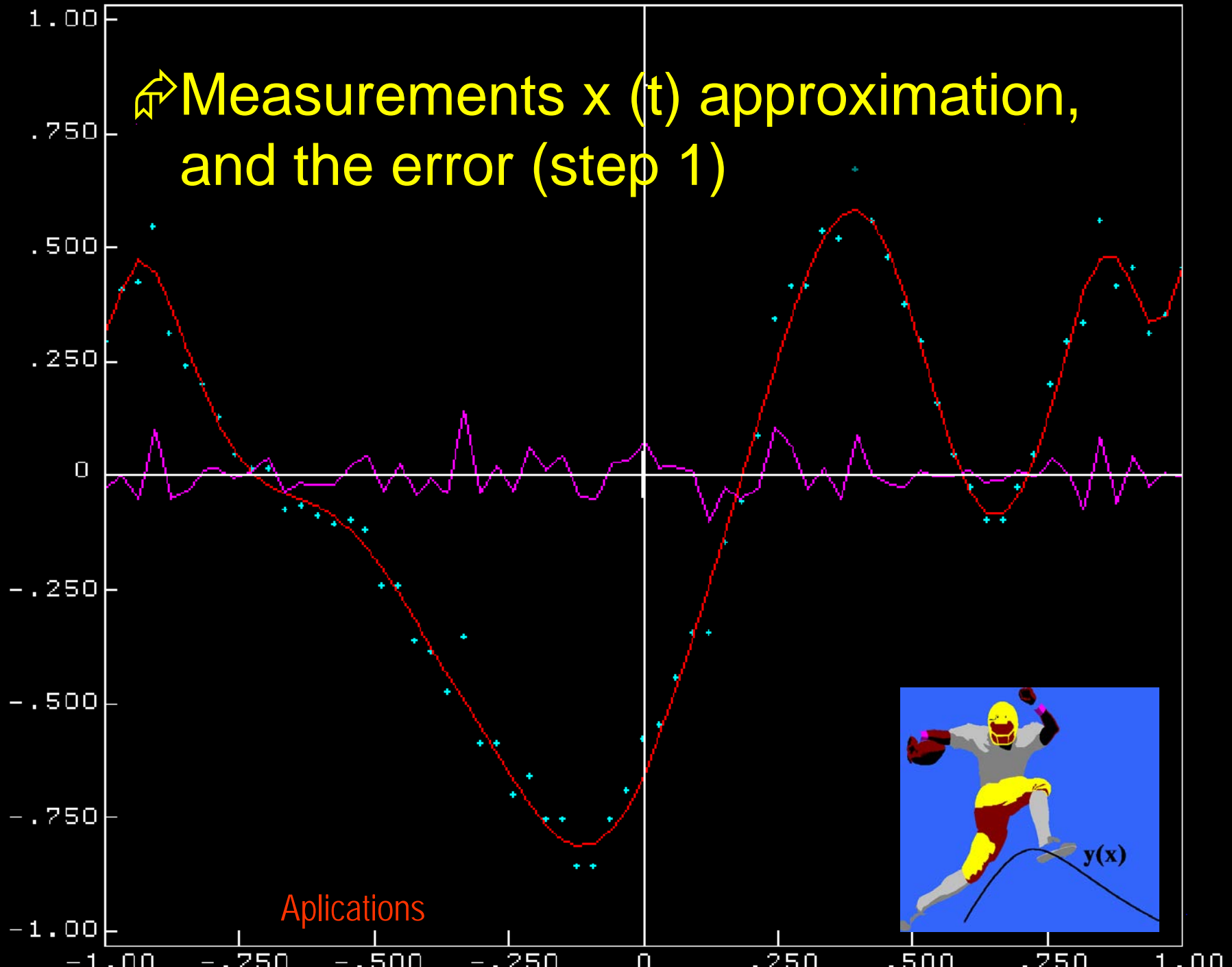
# Biomechanical data analysis

□ Kinematics of human body

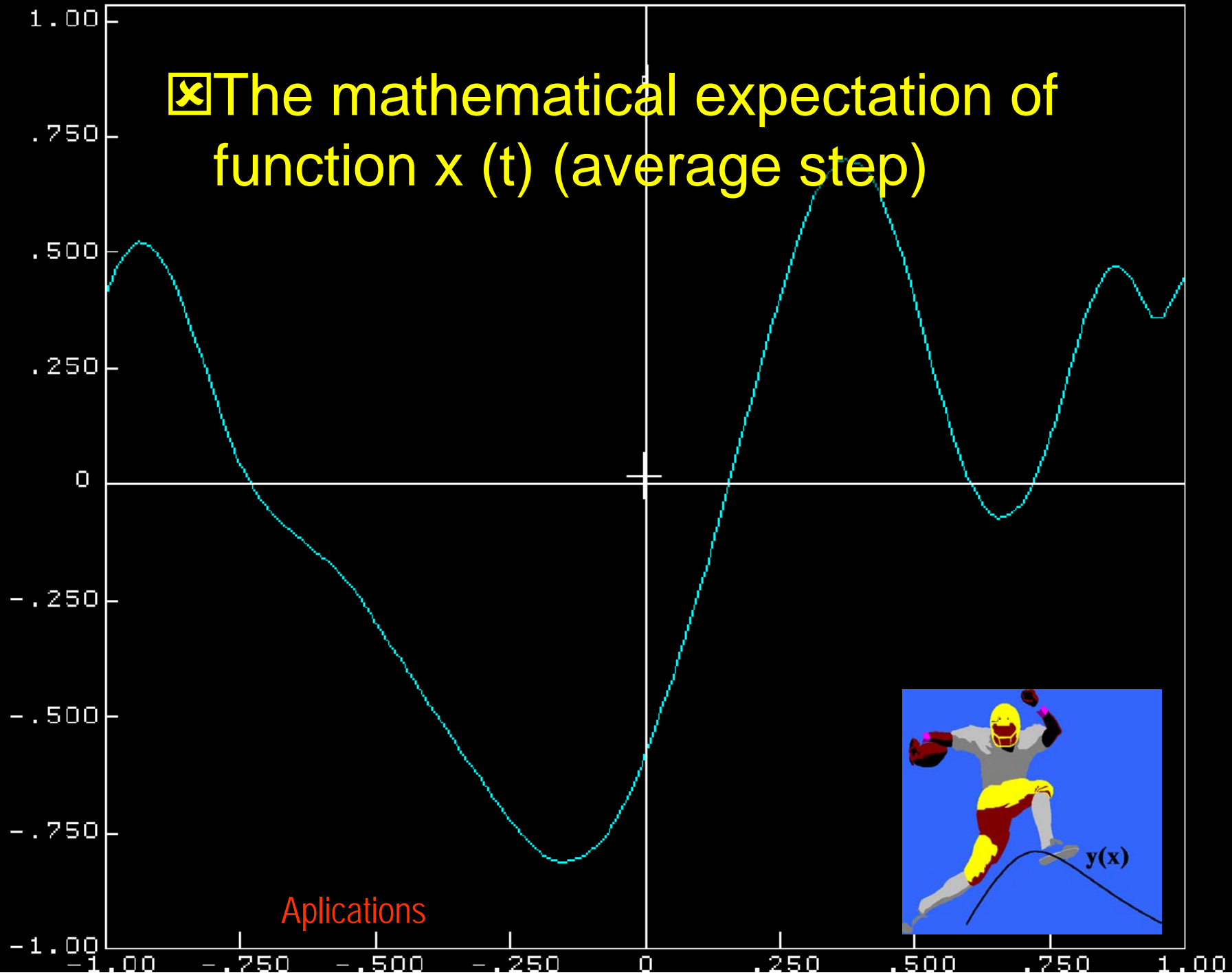
Applications



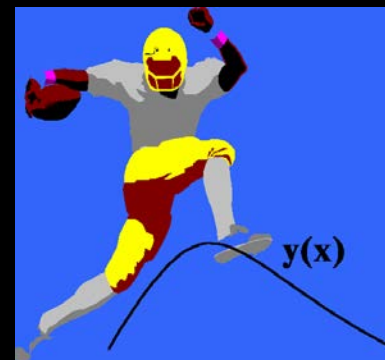
# Measurements $x(t)$ approximation, and the error (step 1)



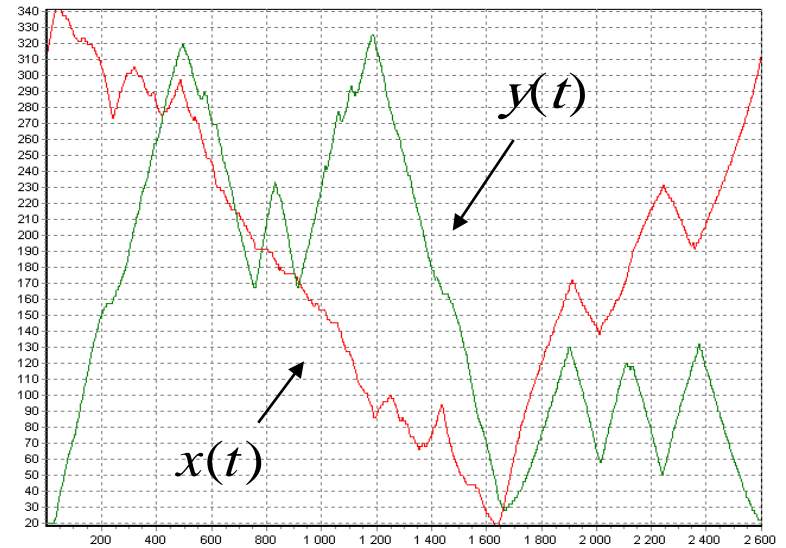
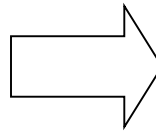
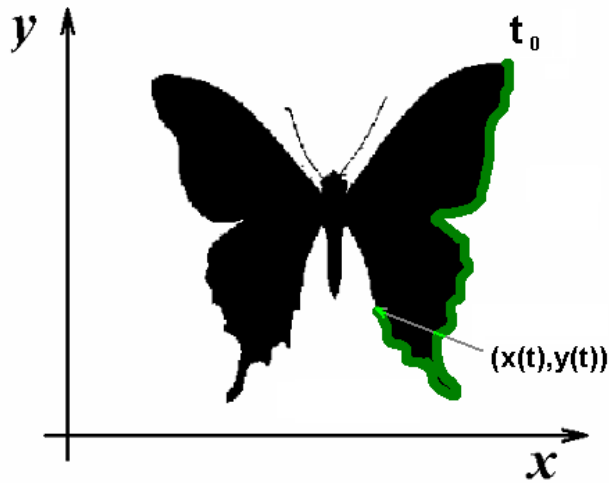
✘ The mathematical expectation of function  $x(t)$  (average step)



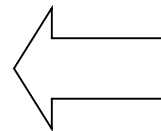
Applications



# Example of parametric description

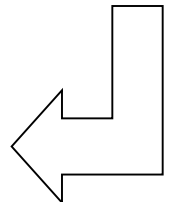


	0	1	2	3	4	...	
$A_n$	91.2	568	141	-148	-30		$\sum_{n=1}^N \sqrt{A_n^2 + B_n^2}$
$B_n$	76.6	-318	342	-107	-68		
$\sqrt{A_n^2 + B_n^2}$		651	370	182	74.7		1317

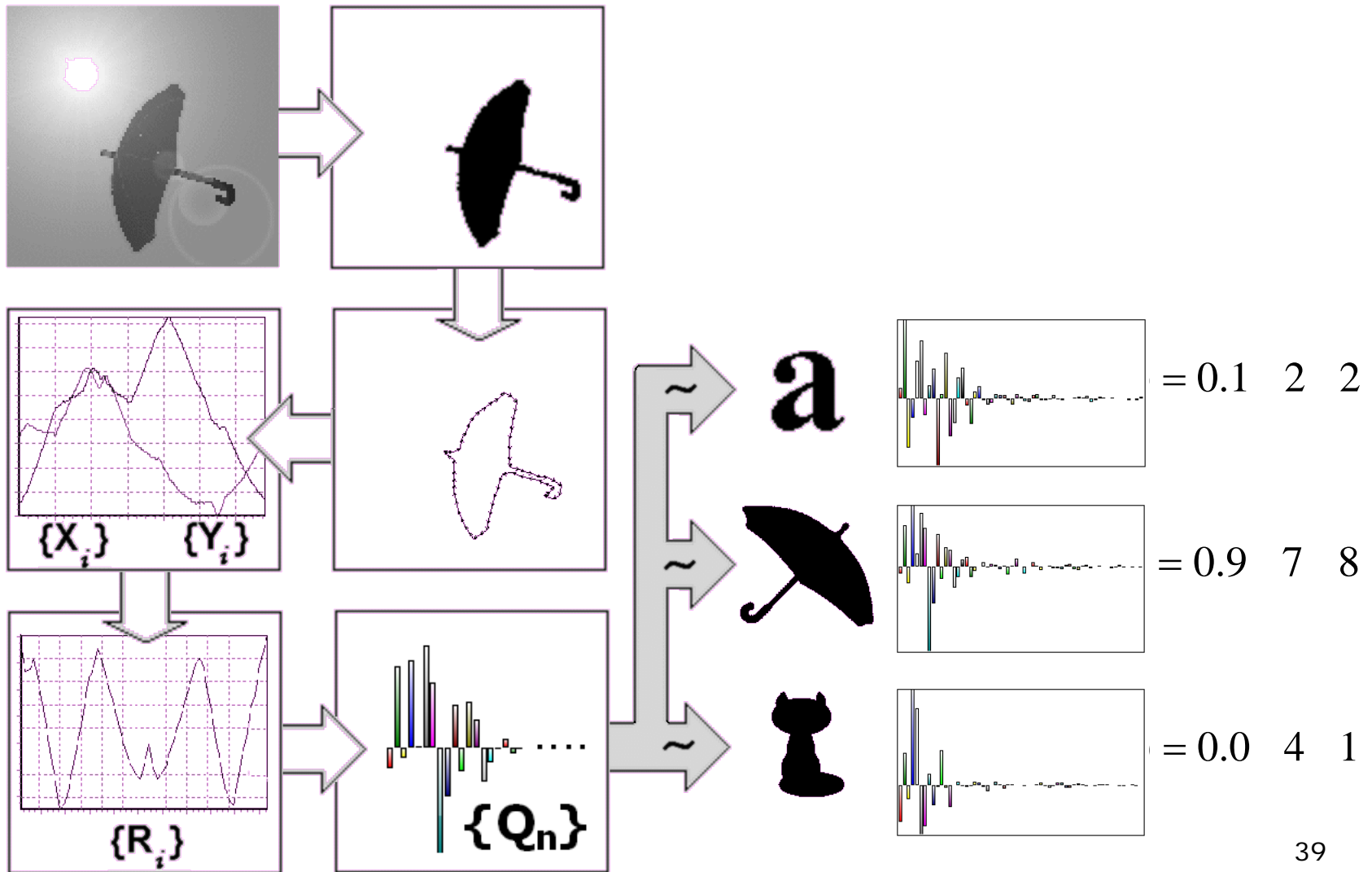


$$x(t) = \sum_{n=0}^N A_n \varphi_n(t);$$

$$y(t) = \sum_{n=0}^N B_n \varphi_n(t).$$



# The general scheme of objects comparison



# Coclusions

The information technology solves the following tasks:

- analytical description of the different nature signals
- spectral data conversion using the developed mathematical libraries
- systems assessment and recognition of abnormal behavior
- time series analysis and the search for sites with desired properties
- noise filtering, direct and reverse integral-differential problems solving



*Děkuji za pozornost!*